Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					

A	Q	A	
/ 1			

General Certificate of Education Advanced Level Examination June 2014

# **Mathematics**

**MM03** 

For Examiner's Use

Examiner's Initials

Mark

Question

1

2

3

4

5

6

7

**TOTAL** 

Unit Mechanics 3

Friday 6 June 2014 1.30 pm to 3.00 pm

#### For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

### Time allowed

• 1 hour 30 minutes

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- · Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### **Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.





## Answer all questions.

Answer each question in the space provided for that question.

A tennis ball is projected from a point O with a velocity of  $(4\sqrt{3}\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ , where  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal and vertical unit vectors respectively. The ball travels in a vertical plane through O which is  $30\,\mathrm{cm}$  above the horizontal surface of a tennis court. During its flight, the horizontal and upward vertical distances of the ball from O are x metres and y metres respectively.

Model the ball as a particle.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$$

[4 marks]

(b) The ball hits a vertical net at a point A. The net is at a horizontal distance of  $4 \,\mathrm{m}$  from O.

Determine the height of the point A, above the surface of the tennis court. Give your answer to the nearest centimetre.

[2 marks]

(c) State a modelling assumption, other than the ball being a particle, that you need to make to answer this question.

[1 mark]

REFERENCE	



QUESTION PART REFERENCE	Answer space for question 1



A rod, of length x m and moment of inertia I kg m<sup>2</sup>, is free to rotate in a vertical plane about a fixed smooth horizontal axis through one end.

When the rod is hanging at rest, its lower end receives an impulse of magnitude  $J\,\mathrm{Ns}$ , which is just sufficient for the rod to complete full revolutions.

It is thought that there is a relationship between J, x, I, the acceleration due to gravity  $g\,\mathrm{m\,s^{-2}}$  and a dimensionless constant k, such that

$$J = kx^{\alpha} I^{\beta} g^{\gamma}$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are constants.

Find the values of  $\alpha$ ,  $\beta$  and  $\gamma$  for which this relationship is dimensionally consistent. **[6 marks]** 

QUESTION PART REFERENCE	Answer space for question 2



QUESTION PART REFERENCE	Answer space for question 2



3		A particle of mass $0.5\mathrm{kg}$ is moving in a straight line on a smooth horizontal su	ırface.
		The particle is then acted on by a horizontal force for 3 seconds. This force at the direction of motion of the particle and at time $t$ seconds has magnitude $(3t+1){ m N}.$	cts in
		When $t=0$ , the velocity of the particle is $4\mathrm{ms^{-1}}$ .	
(a	)	Find the magnitude of the impulse of the force on the particle between the time $t=0$ and $t=3$ .	es
			marks]
(b	)	Hence find the velocity of the particle when $t=3.$	? marks]
(с	)	Find the value of $t$ when the velocity of the particle is $20\mathrm{ms^{-1}}$ .	marks]
QUESTION PART REFERENCE	Ans	wer space for question 3	



QUESTION PART REFERENCE	Answer space for question 3



- Two boats, A and B, are moving on straight courses with constant speeds. At noon, A and B have position vectors  $(\mathbf{i}+2\mathbf{j})\,\mathrm{km}$  and  $(-\mathbf{i}+\mathbf{j})\,\mathrm{km}$  respectively relative to a lighthouse. Thirty minutes later, the position vectors of A and B are  $(-\mathbf{i}+3\mathbf{j})\,\mathrm{km}$  and  $(2\mathbf{i}-\mathbf{j})\,\mathrm{km}$  respectively relative to the lighthouse.
  - (a) Find the velocity of A relative to B in the form  $(m\mathbf{i} + n\mathbf{j}) \operatorname{km} h^{-1}$ , where m and n are integers.

[4 marks]

(b) The position vector of A relative to B at time t hours after noon is  $\mathbf{r} \, \mathrm{km}$ .

Show that

$$\mathbf{r} = (2 - 10t)\mathbf{i} + (1 + 6t)\mathbf{j}$$

[3 marks]

(c) Determine the value of t when A and B are closest together.

[5 marks]

(d) Find the shortest distance between A and B.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 4
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 4



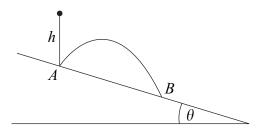
QUESTION PART REFERENCE	Answer space for question 4



QUESTION PART REFERENCE	Answer space for question 4



A small smooth ball is dropped from a height of h above a point A on a fixed smooth plane inclined at an angle  $\theta$  to the horizontal. The ball falls vertically and collides with the plane at the point A. The ball rebounds and strikes the plane again at a point B, as shown in the diagram. The points A and B lie on a line of greatest slope of the inclined plane.



(a) Explain whether or not the component of the velocity of the ball parallel to the plane is changed by the collision.

[2 marks]

(b) The coefficient of restitution between the ball and the plane is e.

Find, in terms of h,  $\theta$ , e and g, the components of the velocity of the ball parallel to and perpendicular to the plane immediately after the collision.

[3 marks]

(c) Show that the distance AB is given by

$$4he(e+1)\sin\theta$$

[7 marks]

QUESTION PART REFERENCE	



QUESTION PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5

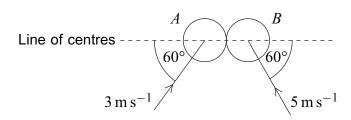


QUESTION PART REFERENCE	Answer space for question 5



Two smooth spheres, A and B, have equal radii and masses  $2 \,\mathrm{kg}$  and  $4 \,\mathrm{kg}$  respectively.

The spheres are moving on a smooth horizontal surface and collide. As they collide, A has velocity  $3\,\mathrm{m\,s^{-1}}$  at an angle of  $60^\circ$  to the line of centres of the spheres, and B has velocity  $5\,\mathrm{m\,s^{-1}}$  at an angle of  $60^\circ$  to the line of centres, as shown in the diagram.



Just after the collision, *B* moves in a direction perpendicular to the line of centres.

(a) Find the speed of A immediately after the collision.

[6 marks]

(b) Find the acute angle, correct to the nearest degree, between the velocity of A and the line of centres immediately after the collision.

[2 marks]

**(c)** Find the coefficient of restitution between the spheres.

[2 marks]

**(d)** Find the magnitude of the impulse exerted on *B* during the collision.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6



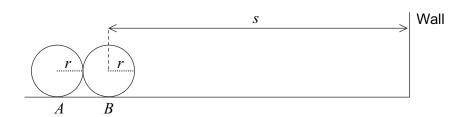
- 7 Two small smooth spheres, A and B, are the same size and have masses 2m and m respectively. Initially, the spheres are at rest on a smooth horizontal surface. The sphere A receives an impulse of magnitude J and moves with speed 2u directly towards B.
  - (a) Find J in terms of m and u.

[2 marks]

(b) The sphere A collides directly with B. The coefficient of restitution between A and B is  $\frac{2}{3}$ . Find, in terms of u, the speeds of A and B immediately after the collision.

[5 marks]

(c) At the instant of collision, the centre of B is at a distance s from a fixed smooth vertical wall which is at right angles to the direction of motion of A and B, as shown in the diagram.

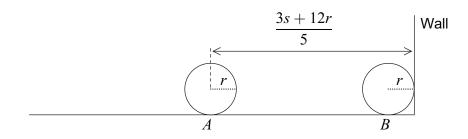


Subsequently, B collides with the wall. The radius of each sphere is r.

Show that the distance of the centre of A from the wall at the instant that B hits the wall is  $\frac{3s+12r}{5}$ .

[4 marks]

(d) The diagram below shows the positions of A and B when B hits the wall.



The sphere B collides with A again after rebounding from the wall. The coefficient of restitution between B and the wall is  $\frac{2}{5}$ .

Find the distance of the **centre of**  $\boldsymbol{B}$  from the wall at the instant when A and B collide again.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 7



QUESTION PART REFERENCE	Answer space for question 7



QUESTION PART REFERENCE	Answer space for question 7
	END OF QUESTIONS



